

Editorial

Dear Reader

Many researchers have dealt with, and continue to deal with, problem solving, definitions of the notion of a problem, the roles of problem solving in mathematics with regard to the development of procedural and conceptual knowledge, and differentiating between investigation and problem solving. In general, a problem in mathematics is defined as a situation in which the solver perceives the situation as a problem and accepts the challenge of solving it but does not have a previously known strategy to do so, or is unable to recall such a strategy. The best known strategies in mathematics are inductive reasoning and deductive reasoning. Inductive reasoning involves, on the basis of observations of individual examples, deriving a generalisation with a certain level of credibility (in mathematics, if the generalisation is not proven we must not take it as true, as always applicable). With deductive reasoning, on the other hand, we derive examples on the basis of a broadly accepted generalisation that serve to illustrate the generalisation. Both forms of reasoning are important in mathematical thinking. In addition to these two forms of reasoning – deductive and inductive – certain authors in the field of mathematics use other collocations, such as inductive inference, and reasoning and proof. In the majority of cases, researchers investigating problem solving associate the issues of problem solving with inductive reasoning. Research in the field of problem solving is focused on the cognitive processes associated with strategies used by enquirers (students of all levels) in solving selected problems, as well as on the significance of inductive reasoning for the development of the basic concepts of algebra. By analysing the process of solving problems, we gain insight into the strategies used by solvers, on the basis of which we can draw conclusions about the success of specific strategies in forming generalisations. An important finding in this regard is that not all strategies are equally efficient, and that the context of a problem can either support or hinder generalisation. However, selecting a good mathematical problem is not the only criteria for successful generalisation. Another important factor is the social interaction between the solvers of the problem, which means that when solving a problem, in addition to the dimension subject-object (solver-problem), it is also necessary to take into account the dimension subject-subject (solver-solver, solver-teacher).

The foundations of problem solving in mathematics instruction were established by Polya, who identified four levels of inductive reasoning within problem solving: observation of particular cases, conjecture formulation, based

on previous particular cases, generalization and conjecture verification with new particular cases. Other researchers of problem solving have added further levels, such as: organization of particular cases, search and prediction of patterns, conjecture formulation, conjecture validation and general conjectures justification. It is rare that all of the levels are present when solving a particular example, as problem solving it is primarily dependent on the solver, on his/her knowledge and motivation, as well as on other factors.

The teacher plays an important role in problem solving in school, with knowledge, problem selection and the way the problem situation is conveyed, as well as by guiding students through the process of solving the problem. The greater the teacher's competence in problem solving, the greater the likelihood that s/he will include problem situations in mathematics instruction, and thus develop this competence in the students. A leading group in the field of researching problem-solving instruction and the inclusion of problem solving in mathematics instruction is ProMath, whose aim is to examine mathematical-didactical questions concerning problem solving in mathematics education. The group was formed on the initiative of Prof. Dr Günter Graumann (University of Bielefeld, Germany), Prof. Dr Erkki Pehkonen (University of Helsinki, Finland) and Prof. Dr Bernd Zimmermann (University of Jena, Germany). Originally founded in 1998 as a Finnish-German group, it has developed into an international collaboration with an European focus.

In the present edition of the CEPS Journal, members of ProMath have highlighted various issues regarding problem solving, from a reconsideration of the meaning and role of problem solving and a study of the factors determining successful problem solving, to the use of ICT in problem solving.

In the first article, *On Teaching Problem Solving in School Mathematics*, Erkki Pehkonen, Liisa Näveri and Anu Laine present an overview of the situation in the field of problem solving, as well as outlining the key activities and goals of the ProMath research group, whose aim is to improve mathematics instruction in school. They emphasise the importance of open problems in primary school education, as well as the role of the teacher in developing methods of instruction that include solving mathematical problems.

Benjamin Rott's contribution, *Process Regulation in the Problem-Solving Processes of Fifth Graders*, investigates how problem solving processes take place amongst fifth graders, as well as examining the influence of metacognition and self-regulation on these processes and whether transitions between the phases in the process of problem solving are linked with metacognitive activities. On the basis of an analysis of approximately one hundred fifth graders (aged 10–12 years) in German primary schools, the author concludes that there

is a strong link between processes regulation and success (or lack of success) in problem solving.

The third article is by Ana Kuzle and is entitled *Promoting Writing in Mathematics: Prospective Teachers' Experiences and Perspectives on the Process of Writing When Doing Mathematics as Problem Solving*. It reports on research focused on gaps between writing and mathematical problem solving in instruction. At a problem-solving seminar, preservice teachers gained experience in writing in mathematics, and reported on how the experience influenced the process of problem solving and formed their attitude towards including writing in their own lessons. Those who perceived writing and mathematics instruction as one interwoven process expressed a positive attitude towards the use of writing in mathematics lessons, whereas those who viewed writing and mathematics instruction as two separate processes used writing purely as a method to create a formal document in order to satisfy the demands of teachers.

In an article entitled *Applying Cooperative Techniques in Teaching Problem Solving*, Krisztina Barczy presents cooperative learning as one way of overcoming the difficulty students face in making the transition from simple mathematical tasks to solving mathematical problems. The article describes the positive effects of the cooperative teaching techniques in a group of secondary school students aged from 16 to 17 years. These effects include a greater willingness amongst the students to share their opinions with other members of the group, and the development of independent thinking.

In the fifth article, *Improving Problem-Solving Skills with the Help of Plane-Space Analogies*, László Budai focuses on students' problems in dealing with the geometrical treatment of three-dimensional space. The author identifies the possibility of improving the situation in this field by creating teaching procedures that strengthen analogies between planar and spatial geometry. The article demonstrates the important role of the geometry programmes *GeoGebra* and *DGS* in developing spatial awareness and solving spatial geometry problems amongst secondary school students.

Zlatan Magajna's contribution, *Overcoming the Obstacle of Poor Knowledge in Proving Geometry Tasks*, presents one option for more successfully proving claims regarding planar geometry. Proving a claim in planar geometry involves several processes, the most salient being visual observation and deductive argumentation. These two processes are interwoven, but often poor observation hinders deductive argumentation. The article presents the results of two small-scale research projects involving secondary school students, both of which indicate that students are able to work out considerably more deductions if computer-aided observation is used. However, not all students

use computer-aided observation effectively, as some are unable to choose the properties that are relevant to the task from the exhaustive list of properties observed by the computer programme.

In the *Varia* section we find one paper by Nada Turnšek, entitled *Enjoying Cultural Differences Assists Teachers in Learning about Diversity and Equality. An Evaluation of Antidiscrimination and Diversity Training*. In the paper a study based on a quasi-experimental research design is presented. The results of an evaluation of Antidiscrimination and Diversity Training that took place at the Faculty of Education in Ljubljana, rooted in the anti-bias approach to educating diversity and equality issues showed that ADT had a decisive impact on all of the measured variables. It was also found that self-assessed personality characteristics are predictors of the teachers' beliefs, especially the *enjoying awareness of cultural differences* variable, which correlates with all of the dependent variables.

In the last section a review by Darko Štrajn of a monograph *The Globalisation Challenge for European Higher Education / Convergence and Diversity, Centres and Peripheries*, edited by Zgaga, P., Teichler, U., and Brennan, J. (2012, Frankfurt/M.: Peter Lang. ISBN 978-3-631-6398-5) is presented. As stated in the review the editors and authors of the book, which is an insightful product of a range of institutionally and informally based academic interactions, were obviously aware that the developments in European higher education systems expose a chain of meanings to different perceptions and to critical scrutiny.

TATJANA HODNIK ČADEŽ AND VIDA MANFREDA KOLAR