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Enhancing Mathematical Creativity Through a 21st-Century-Based Learning Model: A Mixed Methods Study

Alimuddin Tampa*[1], Ja'faruddin[2], Fauziyyah Alimuddin[2] and Andi Mulawakkan Firdaus[3]

The present study investigates the effectiveness of the 21st-Century-Based Mathematics Learning Model in enhancing junior high school students' mathematical creativity on a large scale. A quasi-experimental design with a mixed-methods approach was employed, involving 369 students from 12 schools located in urban, suburban and rural areas of South Sulawesi, Indonesia. The primary instrument was an open-ended mathematical creativity test designed to measure students' fluency and flexibility in problem solving. Quantitative data were collected through pretests and posttests and analysed using descriptive statistics, paired-samples t-tests, Kruskal–Wallis tests and effect size calculations (Cohen's *d* and *N*-gain). Qualitative data were obtained through classroom observations and analysed using thematic analysis to explore patterns of originality in students' creative thinking processes. The results indicate significant improvements in fluency and flexibility (Cohen's d = 3.64; N-gain = 0.50), with significant differences based on school location, where urban students demonstrated higher performance. The qualitative findings reveal emerging patterns of originality in students' questioning behaviours and mathematical communication. The 21st-Century-Based Mathematics Learning Model integrates structured learning syntax, a collaborative social system and reflective dialogue, all of which support 21st-century competencies. The present study provides empirical evidence of the model's effectiveness across diverse contexts and offers methodological implications for the development of context-sensitive instructional innovations.

Keywords: mathematical creativity, 21st-Century-Based Mathematics Learning Model, 21st-century learning framework, geographical variation, mixed methods research

1 *Corresponding Author. Faculty of Mathematics and Natural Sciences, Makassar State University, Indonesia; alimuddin@unm.ac.id.
2 Faculty of Mathematics and Natural Sciences, Makassar State University, Indonesia.
3 Faculty of Teacher Training and Education, Universitas Muhammadiyah Makassar, Indonesia.

Krepitev matematične ustvarjalnosti s pomočjo modela za učenje, zasnovanega za 21. stoletje: študija z mešanimi metodami

Alimuddin Tampa, Ja'faruddin, Fauziyyah Alimuddin in Andi Mulawakkan Firdaus

Ta študija preučuje učinkovitost modela za učenje matematike, zasnovanega za 21. stoletje, pri krepitvi matematične ustvarjalnosti učencev zadnjega triletja osnovne šole v širšem obsegu. Uporabili smo kvazieksperimentalni načrt s pristopom mešanih metod, v katerega je bilo vključenih 369 učencev iz 12 šol v mestnih, predmestnih in v podeželskih območjih južnega dela Sulavezija v Indoneziji. Glavni instrument je bil odprt test matematične ustvarjalnosti, zasnovan za merjenje spretnosti in prilagodljivosti učencev pri reševanju problemov. Kvantitativni podatki so bili zbrani s predtestiranjem in potestiranjem ter analizirani z uporabo deskriptivne statistike, t-testov za pare vzorcev, Kruskal-Wallisovih testov in izračunov velikosti učinka (Cohenov *d* in *N*-gain). Kvalitativni podatki so bili pridobljeni z opazovanjem v razredih in analizirani s pomočjo tematske analize, da bi raziskali vzorce izvirnosti v procesih kreativnega mišljenja učencev. Izsledki kažejo na znatno izboljšanje spretnosti in prilagodljivosti (Cohenov d = 3,64; N-gain = 0,50), pri čemer so bile ugotovljene znatne razlike glede na lokacijo šole, saj so se učenci iz mestnih območij bolje odrezali. Kvalitativne ugotovitve razkrivajo nastajajoče vzorce izvirnosti v vprašujočem vedenju učencev in matematični komunikaciji. Model za učenje matematike, zasnovan za 21. stoletje, združuje strukturirano učenje, sistem sodelovanja in socialne interakcije ter refleksivni dialog, kar vse podpira kompetence 21. stoletja. Ta študija prinaša empirične dokaze o učinkovitosti modela v različnih kontekstih in ponuja metodološke implikacije za razvoj inovativnih pedagoških pristopov, prilagojenih posameznim kontekstom.

Ključne besede: matematična ustvarjalnost, model za učenje matematike, zasnovan za 21. stoletje, okvir učenja za 21. stoletje, zemljepisne razlike, raziskava z mešanimi metodami

Introduction

Amid global competition and rapid technological advancement, teacher education reform emphasises the need to align pedagogy with digital transformation and global challenges (Perla et al., 2024). The demand for higher-order thinking skills, particularly mathematical creativity, continues to increase (OECD, 2018; Beswick & Fraser, 2019). Mathematical creativity is understood as the ability to generate novel ideas and solve complex problems in flexible and innovative ways, a competence that is crucial for junior high school students when addressing real-world problems (Al Moray, 2024; Cevikbas et al., 2022).

Numerous studies indicate that students' mathematical creativity in open-ended tasks remains low. Students tend to produce a single solution, rely heavily on routine procedures, and demonstrate limited fluency and flexibility in their strategies (Nurkaeti et al., 2020; Tanudjaya & Doorman, 2020). These findings are consistent with the preliminary analysis of the present study, which shows that in a sample of 63 students, 97.3% produced only one or no solution, while only 2.7% were able to generate two correct solutions.

One major contributing factor is the dominance of one-way instructional approaches (Hu & Wang, 2024; Bicer et al., 2024). Although problem-based learning, discovery learning and project-based learning have been widely adopted, creativity is often positioned as an implicit outcome rather than an explicit instructional objective. Students are not systematically supported to transform problems into multiple representations, compare alternative strategies or revise ideas through critical feedback. As a result, the development of mathematical creativity remains limited and sporadic (Milin Šipuš et al., 2021; Marwa et al., 2024; Mariani et al., 2025). This condition underscores the need for instructional interventions that explicitly and systematically foster mathematical creativity. In response to this need, the authors have developed and validated the 21st-Century-Based Mathematics Learning (21st-BML) model.

Mathematical Creativity

Mathematical creativity refers to the ability to generate diverse and adaptive solutions by integrating convergent and divergent thinking (Kwon et al., 2025). Schoevers et al. (2019) distinguish professional creativity in research contexts from school-level creativity, which emphasises students' exploration of problem-solving processes. Within the Guilford–Torrance framework, mathematical creativity comprises three indicators: fluency, flexibility and originality (Guilford, 1967; Torrance, 1974; Bingol & Ozyaprak, 2025). These indicators show different measurement characteristics (Leikin & Sriraman, 2017): fluency and flexibility can be operationalised through the number of valid solutions and the diversity of strategies, whereas originality is contextual because it depends on the perceived uncommonness of solutions (Schoevers et al., 2019). In school settings, an idea may be uncommon for students even if it is not mathematically original, which makes originality difficult to assess objectively through written tests alone.

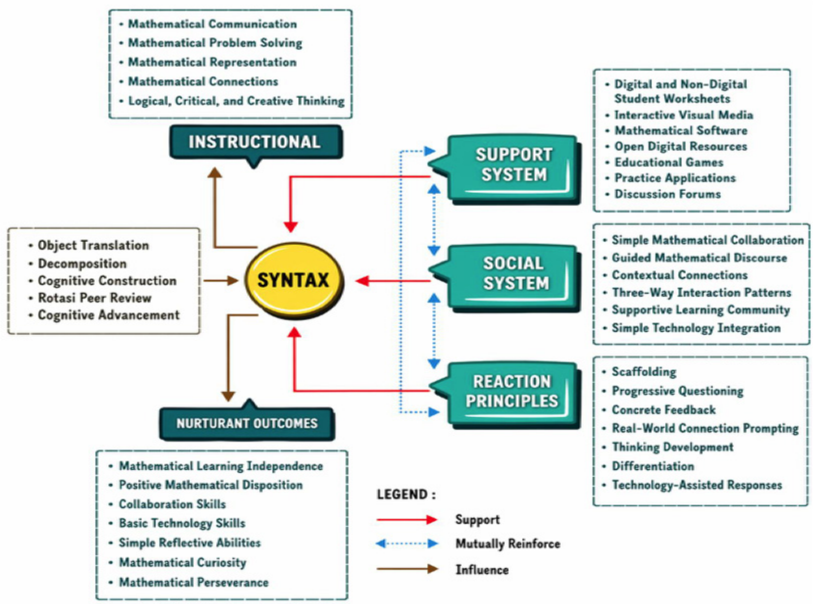
Accordingly, prior studies often separate approaches, using quantitative analyses for fluency and flexibility, while qualitative analyses is used for originality based on observations, student artifacts and verbal interactions. Mathematical creativity is also viewed as a process evident in students' engagement in problem solving, problem posing and mathematical communication across representations (Silver, 1997; Sfard, 2008; Sitorus & Masrayati, 2016), shaped by task design, social interaction and learning autonomy (Leikin, 2020; Wessels, 2014). Therefore, analysing patterns of creative activity is essential for understanding how mathematical creativity develops.

The 21st-BML Model

The 21st-BML model was developed through an R&D study based on the Plomp development model and Nieveen's evaluation criteria (Plomp, 2013; Nieveen, 2010). Prior work indicates that the model meets validity and practicality criteria and can enhance students' mathematical creativity (Tampa et al., 2024). However, the development phase relied on a small-scale trial with one class (32 students) in a relatively homogeneous setting, so the model's empirical base and conceptual structure require further strengthening.

Accordingly, large-scale implementation across diverse contexts (e.g., school location, status and accreditation) is needed to provide more robust evidence. Conceptually, the 21st-BML model comprises five interrelated components that function as an integrated learning system, with their relationships illustrated in Figure 1.

Figure 1
Coherence Among Components of the 21st-BML Model



Learning Framework of the 21st-BML Model

Based on Figure 1, the learning framework of the 21st-BML model (Table 1) summarises the core functions of each learning syntax and is operationalised into instructional materials (lesson plans, student worksheets, learning media, enrichment materials and assessment instruments) as the model's concrete implementation in classroom practice (Tampa et al., 2024).

Table 1
Learning Framework of the 21st-BML Model

Learning Syntax	Description of the Learning Syntax (Functions and General Activities)
1. Object Translation	Students are guided to translate contextual problems into mathematical representations through the process of enactive metaphorising, which bridges concrete experiences with abstract ideas. This process is supported by embodied metaphors, enabling the cognitive transition from iconic to symbolic thinking.
2. Decomposition	Students are directed to organise and connect mathematical concepts through visual mapping and analysis of relationships between elements, thereby forming a coherent structure of understanding.

Learning Syntax	Description of the Learning Syntax (Functions and General Activities)
3. Cognitive Construction	Students are facilitated in building cognitive structures developed through the preceding learning syntax. This is achieved through exploration, elaboration and synthesis of ideas, conducted individually or collaboratively, in completing problem-based student worksheets featuring contextual open-ended tasks.
4. Rotational Peer Review	Students are engaged in exchanging work outcomes and conducting peer evaluations to develop mathematical communication and critical thinking, and to broaden their perspectives on problem solving.
5. Cognitive Advancement	Students extend and integrate prior cognitive structures through open-ended, independent, time-bound explorations based on real-life situations from their environment. Activities involve observation, analysis and creating mathematical models in symbolic, visual, verbal or digital forms, documented with progress reports, online communication and periodic feedback for reflective strategy adjustments.

The Role of the Learning Syntax of the 21st-BML Model in Fostering Mathematical Creativity

Each syntax in the 21st-BML model is designed to support specific dimensions of mathematical creativity. Object Translation supports the transition from contextual experience to abstraction through enactive and embodied metaphors, strengthening fluency and flexibility (Bernadez & Montero, 2025; Bicer, 2021) and aligning with evidence on the role of problem posing and problem solving in fostering flexibility and originality (Papadopoulos et al., 2022). Decomposition develops relational mapping and conceptual organisation, promoting fluency, flexibility and originality (Hamid et al., 2024). Cognitive Construction builds these structures through open-ended, context-based worksheet tasks, supporting strategic synthesis and reinforcing fluency and originality (Kholid et al., 2024; Arifin et al., 2021), consistent with structured learning management principles (Kónya & Kovács, 2021). Rotational Peer Review expands perspectives through peer evaluation and argumentation, enhancing flexibility and originality (Yayuk et al., 2020). Cognitive Advancement extends prior cognitive structures through independent, context-based exploration supported by documentation and feedback, fostering flexibility, fluency and collective creativity (Jonsson et al., 2022; Meier et al., 2023; Ban Hassan Majeed et al., 2023).

Geographical Factors in the Implementation of the 21st-BML Model

School geographical context shapes mathematics learning conditions through differences in resource access, learning culture and institutional support, thereby creating unequal learning opportunities for implementing innovative models (Ma & Liu, 2022; Niu et al., 2025; Zhang et al., 2025). International evidence shows consistent achievement patterns by school location, with urban schools typically outperforming rural schools and suburban schools tending to fall between these contexts, reflecting variations in educational structures and instructional management (Mohammadpour & Yon, 2024; De Nadai et al., 2020).

In Indonesia, similar disparities have been reported, particularly related to technological readiness and teacher quality (Susanta et al., 2025; Baharuddin & Burhan, 2025). More broadly, technological readiness, teacher quality and institutional support vary geographically and may influence the effectiveness of student-centred and collaborative learning models (Li, 2025). Therefore, the present study positions geographical context as a key factor for understanding variation in the implementation and outcomes of the 21st-BML model and for formulating hypotheses regarding location-based differences in effectiveness.

The present study examines the large-scale effectiveness of the 21st-BML model across diverse contexts in order to strengthen its empirical foundation and conceptual structure. It also explores patterns of students' creative activities during implementation to clarify how the model supports mathematical creativity. Accordingly, the study addresses the following research questions:

1. How effective is the 21st-BML model in enhancing lower secondary students' mathematical creativity?
2. Do students' creative gains vary significantly by school location (urban, suburban, rural)?
3. What patterns of creative activity emerge during implementation?

Research Hypotheses

H_1: The 21st-BML model significantly enhances students' mathematical creativity.

H_2: Improvements in students' creativity differ significantly across geographic school types.

Method

Participants

The study involved 369 eighth-grade students from 12 junior high schools in South Sulawesi Province, Indonesia. The sample was selected using stratified cluster random sampling based on school location (urban, suburban, rural), school status (public/private) and accreditation level (A/B). Table 2 presents the participant profile

Table 2

Profile of Quantitative Research Participants

Geographical Location	School Code	School Type	Accreditation Status	Number of Participants	M	F
Urban	KM-1	Public	A	36	15	21
	KM-2	Public	B	34	13	21
	KM-3	Private	A	30	11	19
	KM-4	Private	B	29	9	20
Suburban	PK-1	Public	A	32	11	21
	PK-2	Public	A	32	13	19
	PK-3	Public	B	30	13	17
	PK-4	Private	B	31	12	19
Rural	LK-1	Public	A	32	15	17
	LK-2	Public	A	30	12	18
	LK-3	Public	B	26	9	17
	LK-4	Public	B	27	9	18
Total				**369**	**142**	**227**

Instruments

The primary instrument was a mathematical creativity test (MCT) consisting of three open-ended items on systems of linear equations in two variables (SLETV). The MCT measured fluency (generating multiple valid solutions) and flexibility (using diverse problem-solving strategies). Scoring used an analytic rubric (0–3 for each indicator). Fluency was scored by the number of distinct correct solutions (3 = ≥5; 2 = 3–4; 1 = 1–2; 0 = none), and flexibility by the diversity of the strategies used (e.g., graphical representation, tables, substitution, algebraic manipulation) (3 = ≥3 strategies; 2 = 2; 1 = 1; 0 = unclear). The total score range was 0–18.

The MCT was piloted with 40 students and analysed using the Rasch Model; all items showed acceptable fit (*Infit*/*Outfit* MNSQ). The rubric and scoring examples are provided in the Appendices.

An observation sheet documented creativity-related classroom activities during the implementation of the 21st-BML model, focusing on interactional indicators that are difficult to capture from written documents (spontaneous questions, responses to peers' questions, verbal explanations, argumentation/

rebuttals and collaboration). Originality was identified from observed interactional episodes, especially uncommon questions or arguments relative to the dominant classroom discourse. Two raters independently coded the observation records (87% agreement). The observation sheet is included in the Appendices. The written test rubric was used to score fluency and flexibility only.

Research Design

The study employed a mixed-methods approach to examine mathematical creativity through the implementation of the 21st-BML model (Creswell & Plano Clark, 2018; Tashakkori & Teddlie, 2010). The eight-week study comprised a pretest, six instructional sessions and a posttest using the same instruments. The intervention used six expert-validated 21st-BML instructional packages aligned with the 2013 Curriculum (M = 4.2/5.0), and participating teachers attended a two-day microteaching workshop to support implementation consistency.

Quantitative analysis included Rasch-based instrument checking (eRm package), descriptive statistics, paired-samples t-tests, N-gain calculations, school-location comparisons using the Kruskal–Wallis H test with Mann–Whitney U post hoc tests, and effect size estimation (Cohen's d). Qualitative data comprised group artifacts from 67 collaborative groups (four to five students per group) and observation episodes collected across 72 instructional sessions (six per school). Qualitative data were analysed using a six-phase thematic analysis (Braun & Clarke, 2019), with trustworthiness supported through source triangulation and an audit trail.

For further academic purposes, the complete set of instructional documents is available from the corresponding author upon request.

Results

Quantitative Findings on Students' Mathematical Creativity

Table 3 summarises the descriptive results, showing that the 21st-BML model improved the participating students' mathematical creativity across regions. At pretest, all of the students (100%; n = 369) were in the very low category ($0 \leq X \leq 3$; M = 1.36, SD = 0.92). At posttest, none of the students remained in this category and the mean increased to 9.60 (SD = 2.50), indicating substantial gains in fluency and flexibility (Table 3). Regional patterns showed the greatest improvement in urban schools, followed by suburban and rural schools (Table 3).

Further analyses confirmed these gains. The overall *N*-gain was 0.50 (moderate), with regional values of 0.60 (urban), 0.50 (suburban) and 0.40 (rural) (Table 4). The paired-samples *t*-test indicated a statistically significant improvement with a very large effect size (Cohen's $d = 3.64$; $p < 0.001$) (Table 4). Posttest scores differed significantly by school location (Kruskal–Wallis $H = 129.014$, $p < 0.001$), and post hoc Mann–Whitney *U* tests showed a consistent gradient (urban > suburban > rural) (Table 5). These differences suggest that although the model was effective across contexts, the magnitude of its impact varied with contextual readiness and support.

Table 3.

Comparison of Pretest–Posttest Mathematical Creativity Scores

Region	*N*	Pretest Mean	*SD*	Category	Posttest Mean	*SD*	Category Distribution (%)	Δ Mean
Overall	369	1.36	0.92	Very Low (100%)	9.60	2.50	Low–Moderate: 42 (11.4%); Moderate: 193 (52.3%); High: 134 (36.3%)	+8.24
Urban	129	1.53	0.95	Very Low (100%)	11.50	1.90	Low–Moderate: 7 (5.4%); Moderate: 38 (29.5%); High: 84 (65.1%)	+9.97
Sub urban	125	1.39	0.91	Very Low (100%)	9.30	2.20	Low–Moderate: 18 (14.4%); Moderate: 79 (63.2%); High: 28 (22.4%)	+7.91
Rural	115	1.26	0.89	Very Low (100%)	7.90	2.10	Low–Moderate: 31 (27.0%); Moderate: 69 (60.0%); High: 15 (13.0%)	+6.64

Note. Very Low ($0 \leq X \leq 3$), Low–Moderate ($3 < X \leq 6$), Moderate ($6 < X \leq 10$), High ($10 < X \leq 14$).

Further analyses confirmed these gains (Tables 4 and 5). The overall *N*-gain was 0.50 (moderate), with regional values of 0.60 (urban), 0.50 (suburban) and 0.40 (rural) (Table 4). The paired-samples *t*-test showed a statistically significant improvement with a very large effect size (Cohen's $d = 3.64$; $p < 0.001$) (Table 4). Posttest scores differed significantly by school location (Kruskal–Wallis $H = 129.014$, $p < 0.001$), and post hoc Mann–Whitney *U* tests indicated a consistent gradient (urban > suburban > rural) (Table 5). These findings suggest that although the 21st-BML model was effective across contexts, the magnitude of its impact varied with contextual readiness and support.

Table 4
Effectiveness of the 21st-BML Model: N-Gain and Statistical Significance

Region	*N*-Gain	Category	Mean Difference	*t*(df)	Cohen's *d*
Overall	0.50	Moderate	-8.271	-69.83 (368)***	-3.64
Urban	0.60	Moderate → High			
Suburban	0.50	Moderate			
Rural	0.40	Low–Moderate			

Note. *** $p < 0.001$

Table 5
Regional Differences in Posttest Creativity Scores

Statistical Test	df / Comparison	*H* / *U* Value	*p*-value
Kruskal–Wallis	2	129.014	< 0.001
Mann–Whitney *U*	Urban vs. Suburban	12,530.0	< 0.001
	Urban vs. Rural	13,136.5	< 0.001
	Suburban vs. Rural	9,902.5	< 0.001

These regional differences not only indicate statistical variations in creativity outcomes but also reflect differences in contextual readiness, such as access to resources, teacher competence and institutional support.

Patterns of Students' Creative Activities During the Implementation of the 21st-BML Model

In order to support the representativeness of the qualitative findings, all of the qualitative data were coded across the full dataset. Table 6 summarises the distribution of creativity indicators across group artifacts and classroom interaction episodes. The examples presented in the following sections were selected to represent dominant patterns identified across the dataset rather than isolated cases.

Table 6

Distribution of Mathematical Creativity Indicators Across the Entire Qualitative Dataset

Activity Category	Unit of Analysis	Total Units Analysed	Fluency	Flexibility	Originality	Representative Examples Discussed
Task-Based Mathematical Problem Solving	Group artifacts (student worksheets)	67 groups 378 artifacts	352 artifacts (93%)*	306 artifacts (81%)*	174 artifacts (46%)*	*T-1, T-2, T-3*
Student Questioning	Observed questioning episodes	487 episodes	158 (32%)**	213 (44%)**	291 (60%)**	*P-1 to P-5*
Mathematical Communication	Observed explanation episodes	513 episodes	174 (34%)**	220 (43%)**	282 (55%)**	*J-1 to J-4*

Notes. (1) Percentages indicate the proportion of groups exhibiting the creativity indicator in their artifacts. A single group may demonstrate multiple indicators. (2) Percentages indicate the proportion of activity episodes coded according to the dominant creativity indicator. An episode may involve multiple indicators but is classified based on the most salient feature. (3) Codes *T, P* and *J* refer to activity types and sequences of occurrence rather than to individual students or group identities.

Qualitative analysis indicated three main categories of creative activity: task-based mathematical problem solving reflected in collaborative products, students' spontaneous questioning during classroom interactions, and mathematical communication at individual and group levels.

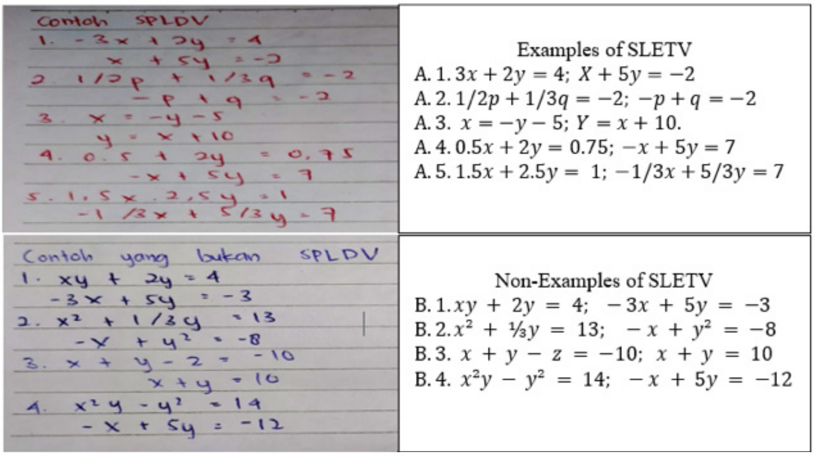
Creativity in Task-Based Mathematical Problem Solving

The students' responses to three representative tasks (*T-1* to *T-3*) showed fluency and flexibility in mathematical problem solving and also indicated emerging originality in how the students constructed and presented strategies. This suggests that 21st-BML tasks support solution and strategy diversity while enabling more unique and contextualised approaches to problem solving.

Data Evidence Task T-1: Examples and Non-Examples of SLETV

Figure 2

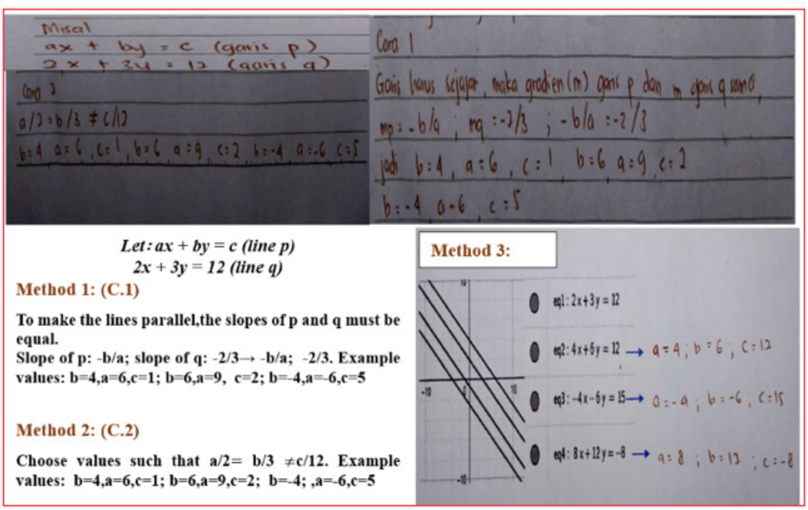
Student Work Sample for Task T-1 on Examples and Non-Examples of SLETV



Data Evidence Task T-2: Inconsistent System of Equations

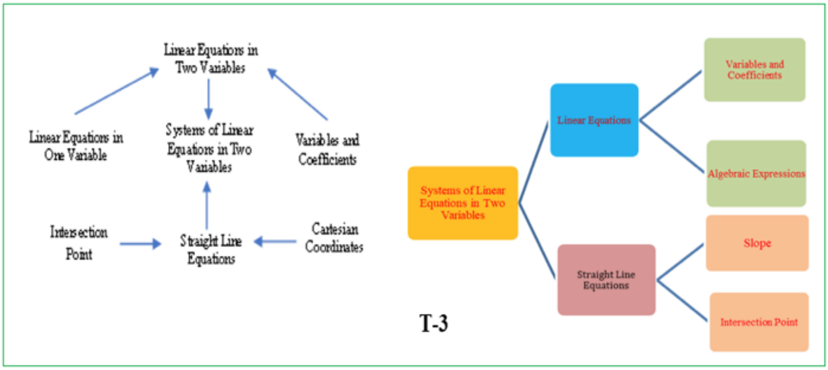
Figure 3

Student Work Sample for Task T-2. Inconsistent System of Equations



Data Evidence Task T-3: Mind Mapping SLETV Equations

Figure 4
Student Work Sample for Task T-3. Mind Mapping SLETV Equations



Fluency was evident across tasks. In *T-1*, the students produced five valid variations of systems of equations (A1–A5) using integer, fractional and decimal coefficients. In *T-2*, they generated multiple parameter sets that yielded inconsistent systems through different constructions. In *T-3*, fluency appeared in the production of varied mind maps that organised concepts related to SLETV.

Flexibility was also consistent. In *T-1*, the students used different construction approaches, including both linear and non-linear forms, to generate diverse systems. In *T-2*, they applied three strategies: gradient-based analysis, algebraic manipulation of constant ratios and visual verification using GeoGebra. In *T-3*, flexibility was reflected in linking algebraic and geometric ideas through different mind-map structures (e.g., radial–hierarchical and modular layouts).

Originality appeared when the students introduced uncommon representations or strategy sequences beyond those modelled in instruction. In *T-1*, this was visible in alternative notations and visual layouts. In *T-2*, the students chose and combined analytical, algebraic and visual strategies in self-directed sequences. In *T-3*, originality emerged in autonomous mind-map organisation, including colour coding, relational structures among concepts and distinctive presentation formats.

Creativity in Questioning and Mathematical Communication

The students' creativity in questioning and communicating mathematical ideas was examined through nine representative expressions: five creative questions (*P-1 to P-5*) and four explanatory responses (*J-1 to J-4*). These expressions illustrate how the students formulated questions and explanations that reflected fluency, flexibility and originality in classroom interaction.

Table 7

Student Expressions of Creativity in Questioning and Communication

Code	Student Quote	Triggering Instructional Phase
P-1	What if we replace the variables in a system of equations with geometric shapes? For example: □ + *Δ* = 10 and *2* □ − *Δ* = 5. Can we calculate the values of □ and *Δ*?	Cognitive Construction
P-2	If the graphs of two equations are parallel, why don't they have a solution? They're both straight and go in the same direction.	Peer Review Rotation
P-3	Do all systems of equations have to be solved with calculations? Can we solve them with pictures or stories instead?	Cognitive Construction
P-4	What if the equations change every second, like in a game or a sensor? Can we still find a fixed solution? And can we make a system where the solution is a fraction, not a whole number?	Cognitive Advancement
P-5	If a store sells orange and apple juice at different prices, how can a system of equations help the owner determine profit?	Peer Review Rotation
J-1	Those two equations are like two roads. If they meet at one point, that means there's one solution that fits both, but if they never meet, then there's no solution.	Peer Review Rotation
J-2	I drew two lines using different colours. If they intersect, I put a circle at that point, because that's the solution.	Peer Review Rotation
J-3	Suppose 3 donuts and 2 teas cost 15 thousand, and 2 donuts and 4 teas cost 16 thousand. We can find the price of one donut and one tea from that.	Cognitive Construction
J-4	Guessing numbers in a game is like solving a system of equations. You need numbers that fit both conditions. If they don't, it's not correct yet. And when I changed $2x + y = 8$ into $y = 8 - 2x$, it felt easier, like rewriting a sentence to make it clearer.	Cognitive Advancement

Note. Code Description: *P* = Creative Questioning; *J* = Creative Communication in Mathematics

Based on Table 7, fluency in questioning was reflected in generating multiple distinct, relevant and mathematically valid questions, as shown in *P-2* and *P-5*, which connected SLETV to parallel-line properties and everyday economic contexts.

Flexibility in questioning was indicated by shifts across representational modes and solution approaches. In *P-1* and *P-3*, the students moved from symbolic forms to visual or narrative representations (e.g., replacing variables with geometric symbols or proposing story-based approaches). Flexibility also appeared in *P-4*, where the students linked SLETV to dynamic technological contexts, shifting from static to dynamic perspectives.

Originality in questioning was evident when the students proposed uncommon ideas not exemplified in instruction, particularly in *P-1* (geometric symbols replacing algebraic variables) and *P-4* (temporally changing systems), while remaining mathematically coherent.

Similar patterns appeared in mathematical communication. Fluency was shown in coherent contextualised explanations (e.g., *J-3* modelling a real-world situation into a system). Communicative flexibility emerged through shifts from symbolic to coloured visual representations and through linking algebraic transformations with linguistic explanation (*J-2* and *J-4*). Originality appeared when the students used uncommon explanatory frames such as analogies and game contexts to explain solution concepts (*J-1* and *J-4*).

Overall, these findings suggest that the Cognitive Construction, Peer Review Rotation and Cognitive Advancement syntaxes provide cognitive and social space for students to generate diverse ideas, shift representations and propose unique questions and explanations through autonomy and exploratory opportunities.

Integration of Quantitative and Qualitative Evidence on Creativity

Integrating quantitative and qualitative results strengthens the interpretation of the 21st-BML model's effectiveness. Quantitative findings show substantial gains in fluency and flexibility, while qualitative evidence explains how these gains were supported through task engagement, questioning and mathematical communication during implementation.

The convergence of evidence indicates that structured task design supports solution diversity and strategy variation, whereas interaction and learning autonomy enable uncommon questions, arguments and explanations to emerge within classroom discourse. Thus, the 21st-BML model enhances both measurable creativity outcomes and the learning processes through which mathematical creativity develops.

Discussion

Effectiveness of the 21st-BML Model in Enhancing Mathematical Creativity

The findings support Hypothesis 1, indicating that the 21st-BML model enhances students' mathematical creativity, particularly fluency and flexibility. This aligns with evidence that innovative approaches such as problem-based learning, flipped classroom, realistic mathematics education, STEM-based learning and open-ended tasks can foster mathematical creativity and especially flexibility and originality (Winasis et al., 2025; Mariani et al., 2025; Khalil et al., 2023; Desi et al., 2025). The added value of the 21st-BML model lies in integrating these influences through five interrelated learning syntaxes. Object Translation and Decomposition support multiple representations and relational organisation of ideas, strengthening fluency and flexibility (Bicer, 2021; Sitorus & Masrayati, 2016). Cognitive Construction and Peer Review Rotation expand idea exploration, synthesis and reflective exchange, stimulating questioning and mathematical communication (Yaniawati et al., 2020). Cognitive Advancement complements this process through autonomous, context-based exploration supported by feedback and technology integration (Nurkaeti et al., 2020; Desi et al., 2025).

Differences in Creativity Outcomes Based on Geographical Location

The findings support Hypothesis 2, showing that the magnitude of the gains varies by school location. These differences can be interpreted through contextual conditions: urban schools generally have stronger technological readiness, more consistent teacher professional development and more established collaborative learning cultures, thus enabling more optimal enactment of the 21st-BML syntaxes. In suburban and rural contexts, constraints in access and training continuity, as well as the students' readiness for discussion-based and open-ended learning, may limit implementation, although gradual development across later sessions suggests adaptive potential. This interpretation is consistent with studies linking location-based differences to learning resources and interaction opportunities (Triana et al., 2024; Nadjet et al., 2024; Kriewaldt et al., 2023) and with evidence that technology-supported collaboration can strengthen creativity where readiness is sufficient (Cisnaulin et al., 2025). Therefore, implementation should be context-sensitive, with stronger scaffolding, phased digital integration and more structured collaborative routines in resource-constrained settings.

Patterns of Students' Creative Activities During the Implementation of the 21st-BML Model

The qualitative findings indicate that creative activity during implementation manifests in task-based problem solving, questioning and mathematical communication, highlighting creativity as a process not fully captured through written tests alone. Task-based work primarily revealed fluency and flexibility through diverse solutions and strategy variation supported by Object Translation and Decomposition. Interactional episodes more strongly highlighted originality when the students modified problems, introduced alternative symbols or used new analogies and metaphors, supported by Cognitive Construction and Peer Review Rotation, while Cognitive Advancement enabled the students to integrate knowledge into new contexts through autonomous exploration. Overall, these patterns suggest complementary roles of the five syntaxes and support evidence that integrative, contextual, collaborative and digitally supported learning can enhance students' creativity (Dilekçi & Karatay, 2023; Rahman et al., 2025; Ritter et al., 2020).

Interpreting the Convergence of Quantitative and Qualitative Evidence on Mathematical Creativity

The convergence of evidence suggests that gains reflect changes in students' engagement in mathematical thinking activities. Quantitative improvements in fluency and flexibility provide outcome indicators, while qualitative evidence clarifies the processes through which students explore meaning, connect representations and develop more personalised mathematical communication. This supports the view that mathematical creativity develops through the interaction of structured instructional design and learning autonomy, mediated by social negotiation of meaning. The theoretical contribution of the 21st-BML model lies in articulating a coherent syntactic framework that connects cognitive transitions, collaborative amplification and autonomous exploration within a unified learning trajectory.

Conclusion

The present study shows that integrating structured learning syntaxes in the 21st-BML model enhances junior high school students' mathematical creativity, and especially improves fluency and flexibility. The model was effective across geographical contexts, although gains were strongest in urban schools and weaker in suburban and rural schools, likely reflecting differences in technological readiness, collaborative learning culture and teacher professional development intensity. The qualitative findings further indicate that creativity develops not only in test performance but also through learning processes, particularly in task-based work, questioning and mathematical communication, where originality emerges in interactional episodes.

Practically, the 21st-BML model provides an adaptive framework for strengthening collaborative learning, integrating technology strategically and adjusting instructional routines to local conditions. In resource-constrained contexts, implementation may require stronger scaffolding, phased digital integration and sustained professional support in order to maintain students' engagement in creative mathematical processes.

The present study does have some limitations. The one-group pretest–posttest design limits causal inference. The intervention duration and single educational level restrict conclusions about long-term effects across developmental stages. The geographical focus on one province limits broader national representation. The absence of a comparative design constrains claims about the model's unique contribution relative to other innovative approaches. Finally, quantitative measurement focused on fluency and flexibility, while originality and other outcomes beyond mathematical creativity (e.g., mathematical dispositions) were not systematically examined.

Guidelines for Further Research

Future studies should:

1. examine the long-term implementation of the 21st-BML model across educational levels to assess sustainability and developmental consistency;
2. test effectiveness across diverse regions of Indonesia to strengthen representativeness and generalisability;
3. explore adaptation beyond mathematics (e.g., science, technology, social sciences) to evaluate cross-domain creativity potential;
4. compare the 21st-BML model with other approaches (e.g., problem-based learning, flipped classroom, realistic mathematics education) to

clarify unique contributions.

5. investigate additional outcomes beyond mathematical creativity, including instructional and nurturant effects.

Ethical Statement

This study received ethical approval from the Institute for Research and Community Service (LP2M), Universitas Negeri Makassar, South Sulawesi, Indonesia. All of the participants provided written informed consent after receiving a clear explanation of the study's objectives, procedures and confidentiality protocols. The entire data collection process was conducted in strict compliance with research ethics standards, with all personal identifiers removed to safeguard participant privacy.

Data Availability Statement

The data generated in the current study are not publicly available due to ethical restrictions. However, anonymised data may be made available from the corresponding author upon reasonable request.

Disclosure Statement

The authors have no conflict of interest to declare.

When preparing the final revision of this article, the authors used ChatGPT (OpenAI), version GPT-5, on 13 March 2026, with the following prompts: (1) condense the manuscript without changing the meaning, results or conclusions; (2) remove repetition; and (3) keep all in-text citations consistent. The authors subsequently reviewed and edited the output as necessary and accept full responsibility for the content and integrity of the publication.

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Appendices

The appendices only provide representative excerpts of the instruments, learning materials and media. The complete documents can be requested from the corresponding author.

Appendix A. Research Instruments

A.1 Mathematical Creativity Test (Sample Item)

Question 1 – Specific Solution

Given the system of linear equations below:

$$ax + by = c$$

$$3x - 2y = 11$$

If $(x, y) = (3, -1)$ is a solution to the system and a, b, c are real numbers, a,b, determine as many pairs (a, b, c) as possible that satisfy the conditions. Explain the reasoning and method you used to obtain the solutions. You may apply different approaches such as substitution, tables, graphs or coefficient manipulation.

A.2 Mathematical Creativity Assessment Rubric

Assessed Dimensions

This rubric evaluates students' mathematical creativity based on two quantitative indicators, as defined operationally in the *Instruments* section.

1. ***Fluency***
   The number of distinct and valid solutions or systems of equations generated by the student in accordance with the problem requirements.
2. ***Flexibility***
   The variety of distinct and valid mathematical approaches, strategies or representations employed by the student in solving the problem.

Scoring Scheme per Item

A. Fluency

Score	Description
3	Produces ≥ 5 distinct and correct solutions/systems
2	Produces 3–4 distinct and correct solutions/systems
1	Produces 1–2 correct solutions/systems
0	Produces no correct solutions/systems

B. *Flexibility*

Score	Description
3	Uses ≥ 3 different strategies or representations (*e.g.*, graphs, tables, substitution, algebraic manipulation)
2	Uses 2 different strategies or representations
1	Uses only 1 strategy or representation
0	Shows no clear or logical strategy

Total Score

- Maximum score per item: 6 (Fluency 3 + Flexibility 3)
- Number of items: 3
- Maximum total score per student: 18
- Minimum score: 0

Mathematical Creativity Score Criteria

Score Range	Category
15–18	Very High
11–14	High
7–10	Moderate
4–6	Low
0–3	Very Low

Illustration of Student Test Result Assessment

Problem

Given a system of linear equations in two variables (SLETV):

$$\begin{cases} ax + by = c \\ 3x - 2y = 11 \end{cases}$$

If $(x, y) = (3, -1)$ is a solution and a, b ≠ 0, determine as many ordered triples (a,b,c) as possible, and explain the method used.

Sample Student Response

Method 1: Table (Algebraic Exploration)

By substituting (3, −1) into $ax + by = c$, we obtain $c = 3a - b$.

The student selects several values of and , then computes the corresponding value of , resulting in the following ordered triples:

(1,1,2), (2,1,5), (3,4,5), (1,−2,5), (4,2,10), (5,5,10)

Method 2: Slope (Geometric Approach)

The equation is rewritten as $y = -\frac{a}{b}x + \frac{c}{b}$.

By selecting several different slopes and ensuring that the line passes through the point (3, –1), the following ordered triples are obtained:

(2,2,4), (6,3,15), (3,6,3), (6,8,10)

Scoring

Aspect	Score	Rationale
Fluency	3	Generates ≥ 5 distinct and valid solution pairs
Flexibility	2	Uses two different approaches: a table (algebraic) and a slope-based (geometric) approach
Total	5/6	Near-maximum score

A.3 Observation Sheet (Excerpt)

Indicators of students' creative activity during the implementation of the 21st-BML model included:

- use of multiple mathematical representations (symbolic, graphical, tabular, verbal);
- spontaneous student questioning;
- verbal reasoning in problem-solving discussions.

Observation Sheet

21st-BML Syntax	Observation Focus	Observation Notes
Object Translation	• Mathematical representations created by students when translating from concrete objects. • Questions posed by students. • Student explanations or methods of communicating ideas.	
Decomposition	• Mind maps created by students. • Mathematical concept components included in the mind maps. • Questions posed by students. • Student explanations or methods of communicating ideas.	
Cognitive Construction	• Student outputs in solving open-ended problems (based on strategy variations). • Student outputs in solving open-ended problems (based on solution variations). • Questions posed by students. • Student explanations or methods of communicating ideas.	

Appendix B. Sample Lesson Plan (Excerpt of Meeting 1 – Object Translation Syntax)

A. Learning Activities	
Object Translation	➢ **Stepwise Animated Video Presentation** • The teacher presents Video 1, showing *Mrs Sari shopping at Toko Buah Segar, buying 5 kg of apples and 4 kg of oranges for a total of Rp 42.000*. • The teacher continues with Video 2, where *Mrs Sari returns the next day and buys 3 kg of apples and 2 kg of oranges for Rp 23.000,-* • Students are instructed to observe both videos carefully while recording key information on a structured observation sheet. ➢ **Initial Group Discussion** • The teacher facilitates a group discussion to help students extract information from both video scenarios. • Students are guided to identify **variables** and **constants** involved in the real-life context. ➢ **Mathematical Representation Transformation** • Each group transforms the information from Video 1 into the first linear equation. • Then, they transform the information from Video 2 into the second linear equation. • The teacher provides adaptive scaffolding using guiding questions such as: o What is unknown in this situation? o How can you express the price of apples and oranges using variables? o What is the relationship between the quantity of fruit and the total cost? ➢ **Concept Formulation: System of Linear Equations in Two Variables** • The teacher guides students to formulate the definition of a system of linear equations in two variables (SLETV) based on the equations they derived. • Students express the concept in two formats: o Verbal definition: *A system of linear equations in two variables is…* o General mathematical form: $ax + by = c$ and $px + qy = r$, where $a, b, p, q \neq 0$ ➢ **Dialogic Interaction and Conceptual Reinforcement** • The teacher provides opportunities for students to ask questions about concepts they do not yet understand. • A triadic interaction strategy is facilitated: o Teacher ↔ Student: Clarification through probing questions o Student ↔ Student: Peer teaching and inter-group discussion o Student ↔ Learning Resources: Use of worksheets and digital references ➢ **Scaffolding-Based Learning Support** • The teacher conducts guided practice to help students reach their zone of proximal development. • Scaffolding is tailored to each group's comprehension level: o Basic Level: Direct assistance with variable identification o Intermediate Level: Strategic hints for forming equations o Advanced Level: Reflective questions for deepening understanding ➢ **Documentation and Transition** • Students document their group work in their individual digital portfolios, including: • Transformation of video scenarios into linear equations • Definition of SLETV (verbal and mathematical) • Reflection on what they learned during the activity

B.2 Sample Student Worksheet (Nasi Kuning Context)

Mas Joko sells nasi kuning (yellow rice) at his food stall called NASI KUNING MACCINI. The menu and price list are presented in the image below.

Step 3: Mathematical Modelling (Symbolic Representation)

Tasks to Complete:

1. Discuss the results of Step 2 within your group's WhatsApp chat.
2. Identify two unknowns in the situation – these will serve as variables x and y.
3. Determine relevant constraints, such as price limits, quantity limits and so on.
4. Formulate a context-based problem that reflects the observed situation.
5. Construct a mathematical model based on the given context (i.e., a system of linear equations in two variables).

Appendix C. Sample Digital Media

Biographical note

Alimuddin Tampa, PhD, is a professor of mathematics education whose area of expertise is mathematics learning oriented toward 21st-century skills, particularly mathematical creativity. He has been a lecturer in the Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Negeri Makassar, Indonesia, since 1988. He earned his doctorate in mathematics education from Universitas Negeri Surabaya, Indonesia, with a research focus on mathematical creativity. His research interests include mathematics education, 21st-century skills in mathematics learning, mathematical creativity, and the development of higher-order thinking skills. He has authored ISBN-registered books, registered intellectual property works, and scholarly articles indexed in Scopus and nationally accredited Indonesian journals. He can be contacted at alimuddin@unm.ac.id.

Ja'faruddin, PhD, is an Associate Professor of Applied Mathematics whose scholarly expertise lies at the intersection of mathematics and culture, with particular emphasis on ethnomathematics. Since 2010, he has served as a faculty member in the Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Negeri Makassar, Indonesia. He obtained his doctoral degree in Applied Mathematics from Tunghai University, Taiwan, where his research focused on the integration of mathematical structures and cultural contexts. His research interests include knot theory, projective geometry, mathematics and culture, ethnomathematics, and mathematics education. He has authored several ISBN-registered books, holds registered intellectual property rights, and has published scholarly articles in Scopus-indexed journals and nationally accredited Indonesian journals. He can be contacted at jafaruddin@unm.ac.id.

Fauziyyah Alimuddin, M.Pd., is an assistant professor in the field of mathematics education in the Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Negeri Makassar, Indonesia. Her research interests include mathematics education, mathematics learning, and the development of learning resources in mathematics. She has authored several scientific articles and contributed to intellectual property works. She can be contacted at fauziyyah.alimuddin@unm.ac.id.

Andi Mulawakkan Firdaus, PhD, is an Associate Professor of Mathematics Education with expertise in foundational mathematics learning, particularly number patterns. Since 2014, he has served as a lecturer in the Mathematics Education Program, Faculty of Teacher Training and Education, Universitas Muhammadiyah Makassar, Indonesia. He earned his doctoral degree in Mathematics Education from Universitas Negeri Surabaya, with a dissertation focusing on number pattern learning based on APOS theory. His research interests include mathematics education, number patterns, APOS theory, mathematical critical thinking, and the development of higher-order thinking skills. He has authored ISBN-registered books, produced intellectual property-protected works, and published scholarly articles in Scopus-indexed journals and nationally accredited journals. He can be contacted at andi.mulawakkan@unismuh.ac.id.